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PENETRATION OF INTENSE PULSED MAGNETIC FIELDS INTO A CONDUCTOR

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The penetration of a pulsed magnetic field into an incompressible conductor is treated with consideration of Joulean heat liberation. Solutions are obtained for the case of penetration of a strongly decreasing magnetic field (significantly exceeding the saturation threshold) into a conductive semispace with planar boundary at constant specific heat and thermal conductivity. It is shown that consideration of the effect of bias current, where the limiting magnetic field is specified in the form of a step function, is of principle significance as regards both surface heating of the conductor and maintenance of intense magnetic fields in experimental equipment with planar boundaries.

It is well known [1, 2] that penetration of an intense magnetic field H(x, t) into a planar incompressible conductor (x > 0) can be described by the equations (in MKS units):

$$-\partial H/\partial x = j + \varepsilon_0\varepsilon_R \partial E/\partial t,\ \partial E/\partial x = -\mu_0\mu_R \partial H/\partial t,\ j = \sigma E,$$
$$\partial Q/\partial t = j^2/\sigma - \partial q/\partial x,\ q = -\lambda\partial\theta/\partial x - \tau_0\partial q/\partial t,\ Q = c_V\theta, \tag{1}$$

where j(x, t) is the volume conduction current density; E(x, t) is the electric field strength; $\varepsilon_0 = 8.85\cdot10^{-12}$ A·sec/(V·m); $\mu_0 = 4\pi\cdot10^{-7}$ V·sec/(A·m); μ_R, ε_R are the relative permutivities, which we assume constant (with either $\mu_R = \varepsilon_R = 1$, or $\mu_R = 1$, $\varepsilon_R = 0$, if we neglect displacement current as compared to conduction current); σ = const is the conductivity of the medium; Q(x, t) is the increment in heat content relative to the state at 0°C; q(x, t) is the thermal flux density; θ(x, t) is the conductor temperature; λ is the thermal conductivity coefficient; τ_0 = const is the thermal flux relaxation time; c_V is the specific heat of the conductor.

We will note that if the characteristic thermal flux relaxation time is large in comparison with the relaxation time τ_0, then $q/\tau_0\partial q/\partial t \gg 1$ and the fifth equation of Eq. (1) transforms to the usual Fourier law $q = -\lambda\partial\theta/\partial x$. And if the thermal flux changes significantly more rapidly than relaxation occurs, then $\partial q/\partial t \gg q/\tau_0$ and the fifth expression of Eq. (1) takes on the form

$$\frac{\partial q}{\partial t} = -\frac{\lambda}{\tau_0}\frac{\partial\theta}{\partial x}. \tag{2}$$

Neglecting displacement current in comparison to conduction current and taking $\tau_0 = 0$, we consider the process of magnetic field penetration into the conducting semispace x > 0 with the following boundary and initial conditions:

$$H(0,\ t) = H_0,\ q(0,\ t) = 0\quad (t > 0); \tag{3}$$
$$H(x,\ 0) = 0,\ Q(x,\ 0) = 0\quad (0 < x < \infty) \tag{4}$$

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(H_0 = const is the intense magnetic field).

It is obvious that system (1) can be reduced by eliminating the functions j, E, q, and θ to a pair of equations

$$\frac{\partial H}{\partial t}=b\frac{\partial^2 H}{\partial x^2},\quad \frac{\partial Q}{\partial t}=\frac{1}{\sigma}\left(\frac{\partial H}{\partial x}\right)^2+k\frac{\partial^2 Q}{\partial x^2}\quad (b=1/\sigma\mu_0,\ k=\lambda/c_V). \tag{5}$$

We will now find a solution to system (3)-(5). It can easily be seen that the self-similar variable is representable in the form $\xi = x/2\sqrt{bt}$. We perform the replacement:

$$H(x,t)=H_0h(\xi),\quad Q(x,t)=\mu_0H_0^2g(\xi)\quad (g(\xi)\geqslant 0).$$

From Eqs. (3)-(5) we have

$$h''=-2\xi h'\quad (0<\xi<\infty),\ h(0)=1,\quad h(\infty)=0. \tag{6}$$

Solving the problem of Eq. (6), we find

$$h(\xi)=1-\Phi(\xi)\quad \left(\Phi(\xi)=\frac{2}{\sqrt{\pi}}\int\limits_0^{\xi}e^{-\tau^2}d\tau\right). \tag{7}$$

Using Eq. (7) in Eq. (5) and considering Eqs. (3) and (4), we have

$$g''+\frac{2b}{k}\xi g'=-\frac{4b}{\pi k}\exp(-2\xi^2)\quad (0<\xi<\infty),\quad g'(0)=0,\quad g(\infty)=0. \tag{8}$$

Solving Eq. (8), we find

$$g(\xi)=A-\frac{4b}{\pi k}\int\limits_0^{\xi}e^{-\frac{b}{k}z^2}\left[\int\limits_0^{z}e^{\left(\frac{b}{k}-2\right)\tau^2}d\tau\right]dz \tag{9}$$

$$\left(A=\sqrt{\frac{4b}{\pi k}}\int\limits_0^{\infty}e^{\left(\frac{b}{k}-2\right)\tau^2}\left[1-\Phi\left(\tau\sqrt{\frac{b}{k}}\right)\right]d\tau\right).$$

Using the well known expressions of [3], we obtain

$$A=\frac{1}{\pi\sqrt{1-\frac{2k}{b}}}\ln\frac{1+\sqrt{1-\frac{2k}{b}}}{1-\sqrt{1-\frac{2k}{b}}}, \tag{10}$$

and considering Eq. (9), we have

$$Q(0,t)=\mu_0H_0^2A. \tag{11}$$

We will compare Eq. (11) with Kidder's well known result [1]:

$$Q(0,t)\approx\mu_0H_0^2\frac{2}{\pi}\ln\left(1+\frac{\pi}{2}\sqrt{\frac{b}{2k}}\right)=\mu_0H_0^2B. \tag{12}$$

In particular, for copper $k/b \simeq 0.009$ [1] and from Eqs. (10) and (12), we find A = 1.7 and B = 1.5.

Let Q_* be the quantity of heat required for heating a unit volume of the semiconductor from its initial temperature to the boiling temperature and its complete evaporation. In the future we will assume that upon absorption of the heat Q_* there occurs a change in the conductivity of the material filling the semispace: material is converted from conductor to dielectric, i.e., the conductivity of the material changes by a law

$$\sigma=\begin{cases}\sigma_0=\text{const} & \text{for}\quad Q<Q_*,\\ 0 & \text{for}\quad Q=Q_*.\end{cases} \tag{13}$$

For example, for copper $Q_* \simeq 4.7\cdot 10^{10}$ J/m³ [4].

It is evident from Eq. (11) that if the magnetic field H_0 is sufficiently large ($H_0 > H_{min} = \sqrt{Q_*/\mu_0 A}$), then the surface upon which the conductivity falls to zero (the phase transition surface), can penetrate into the semispace x > 0 by a law x = X(t).

Neglecting displacement currents and assuming $\tau_0 = 0$, below we will consider the process of penetration of an intense magnetic field H_0 into the semispace $x > 0$ in the presence of the phase transition of Eq. (13), assuming that upon the phase transition surface the conditions

$$H(x, t)|_{x=X(t)} = H_0, \quad Q|_{x=X(t)} = Q_*, \quad \frac{\lambda}{c_V}\frac{\partial Q}{\partial x}\Big|_{x=X(t)} = 0 \tag{14}$$

are satisfied.

Thus, we will find a solution of Eqs. (4), (5), and (14). It can easily be seen that the self-similar variable can be represented in the form $\xi = x/2\sqrt{bt}$, where $b = 1/\sigma_0\mu_0$. On the phase transition boundary $\xi = \alpha$ (where α is an unknown constant). We write the law of phase transition boundary motion in the form

$$x = X(t) = 2\alpha\sqrt{bt} \quad (\alpha = \text{const} \geqslant 0). \tag{15}$$

We take $H(x, t) = H_0 h(\xi)$, $Q(x, t) = \mu_0 H_0^2 g(\xi)$ $(g(\xi) \geq 0)$. We now have

$$h'' = -2\xi h' \quad (\alpha < \xi < \infty),\ h(\alpha) = 1,\ h(\infty) = 0. \tag{16}$$

Solving Eq. (16), we obtain

$$h(\xi) = (1 - \Phi(\xi))/(1 - \Phi(\alpha)). \tag{17}$$

Using Eq. (17) in Eq. (5), and considering Eqs. (4) and (14), we find

$$g'' + \frac{2b}{k}\xi g' = -\frac{4b\exp(-2\xi^2)}{\pi k[1 - \Phi(\alpha)]^2} \quad (\alpha < \xi < \infty),\ g'(\alpha) = 0,\ g(\infty) = 0; \tag{18}$$

$$g(\alpha) = Q_*/\mu_0 H_0^2. \tag{19}$$

Solving Eq. (18), we have

$$g(\xi) = \frac{4b}{\pi k[1 - \Phi(\alpha)]^2}\int\limits_{\xi}^{\infty} e^{-\frac{b}{k}z^2}\left[\int\limits_{\alpha}^{z} e^{\left(\frac{b}{k}-2\right)\tau^2}\,d\tau\right]dz.$$

To determine the unknown constant α, we use Eq. (19). Then

$$\sqrt{\frac{4b}{\pi k}}\,\frac{1}{[1 - \Phi(\alpha)]^2}\int\limits_{\alpha}^{\infty} e^{\left(\frac{b}{k}-2\right)\tau^2}\left[1 - \Phi\left(\tau\sqrt{\frac{b}{k}}\right)\right]d\tau = \frac{Q_*}{\mu_0 H_0^2}. \tag{20}$$

Then solution of Eq. (20) yields α.

If in Eq. (20) $\alpha = 0$, then, calculating the integral and with consideration of Eq. (10), we obtain $A = Q_*/\mu_0 H_0^2$. Hence, if $H_0 = H_{min} = \sqrt{Q_*/\mu_0 A}$, it follows from Eqs. (15) and (20), that the rate of motion of the phase boundary is equal to zero.

In Eq. (20) let $\alpha \to +\infty$. Using the well known [5] asymptote of the error integral $\Phi(\alpha)$ as $\alpha \to +\infty$

$$\Phi(\alpha) = 1 - \frac{1}{\sqrt{\pi}}\,\frac{e^{-\alpha^2}}{\alpha}(1 + o(1)) \tag{21}$$

and the rule for integration of asymptotic expansions, we have

$$\int\limits_{\alpha}^{\infty} e^{\left(\frac{b}{k}-2\right)\tau^2}\left[1 - \Phi\left(\tau\sqrt{\frac{b}{k}}\right)\right]d\tau = \sqrt{\frac{k}{4\pi b}}\int\limits_{2\alpha^2}^{\infty}\frac{e^{-t}}{t}\,dt\,(1 + o(1)). \tag{22}$$

It is known [6] that as $\alpha \to +\infty$

$$\int\limits_{2\alpha^2}^{\infty}\frac{e^{-t}}{t}\,dt = \frac{e^{-2\alpha^2}}{2\alpha^2}(1 + o(1)). \tag{23}$$

Considering Eqs. (21)-(23) and transforming in Eq. (20) to the limit as $\alpha \to +\infty$, we find the limiting value of magnetic field $H_0 = H_{max} = \sqrt{2Q_*/\mu_0}$. We will note that H_{max} is independent of λ. Thus, within the framework of the mathematical model of Eqs. (4), (5), and (14), maintenance of magnetic fields greater than H_{max} is impossible in any experimental device with planar boundaries (for the case $\lambda = 0$, the value of H_{max} was obtained in [4, 7]).

In the above treatment, considering the process of penetration of an impulsive magnetic field into a conductor, we have neglected displacement current as compared to conduction current in Maxwell's equations. As a result, we have certain assertions inadequate for physical experiment. For example, in the problem of Eqs. (3)-(5), $\lim_{t\to+0}\left(-\frac{1}{\sigma}\frac{\partial H}{\partial x}\Big|_{x=0}\right)=\lim_{t\to+0}E(0,t)=+\infty$, although it is physically obvious that this cannot be the case.

Below we will consider the problem of penetration of an impulsive magnetic field into a conductive semispace x > 0 with consideration of the term produced by displacement current.

Thus, assuming for simplicity of calculation that $\lambda = 0$ and taking

$$E(x,0)=0,\ q(x,0)=0\quad(0<x<\infty), \tag{24}$$

we find a solution to the problem of Eqs. (1), (3), (4), and (24) for $\mu_R = \varepsilon_R = 1$.

It can easily be seen that by eliminating the functions j and E, we have the problem

$$\frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}+\sigma\mu_0\frac{\partial H}{\partial t}=\frac{\partial^2 H}{\partial x^2},\ H(0,t)=H_0,\ H(x,0)=\frac{\partial H}{\partial t}\Big|_{t=0}=0\ (0<x<\infty)$$

(where $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the speed of light in vacuo). The solution is known [5]:

$$H(x,t)=\begin{cases}-cH_0\dfrac{\partial}{\partial x}\displaystyle\int_{\frac{x}{c}}^{t}I_0\left(\frac{\sigma}{2\varepsilon_0}\sqrt{y^2-\frac{x^2}{c^2}}\right)\exp\left(-\frac{\sigma}{2\varepsilon_0}y\right)dy & \text{for } t>\dfrac{x}{c},\\ 0 \text{ for } t<\dfrac{x}{c}\end{cases}$$

[$I_0(x)$ is a first order Bessel function of zero order with imaginary argument]. Hence from Eq. (1)

$$E(x,t)=\begin{cases}c\mu_0H_0I_0\left(\dfrac{\sigma}{2\varepsilon_0}\sqrt{t^2-\dfrac{x^2}{c^2}}\right)\exp\left(-\dfrac{\sigma}{2\varepsilon_0}t\right) & \text{for } t\geqslant\dfrac{x}{c},\\ 0 & \text{for } t<\dfrac{x}{c}.\end{cases} \tag{25}$$

If $\tau_0 = 0$, then from Eq. (1) $q(x, t) = 0$. If $\tau_0 \neq 0$, then, writing the fifth equation of system (1) in the form $\partial/\partial t[q\exp((1/\tau_0)\cdot t)] = 0$ and considering Eq. (24), we have $q(x, t) = 0$. Using this equality in Eq. (1) and solving the problem for $Q(x, t)$, we obtain

$$Q(x,t)=\begin{cases}\dfrac{\sigma}{\varepsilon_0}\mu_0H_0^2\displaystyle\int_{\frac{x}{c}}^{t}I_0^2\left(\frac{\sigma}{2\varepsilon_0}\sqrt{y^2-\frac{x^2}{c^2}}\right)\exp\left(-\frac{\sigma}{\varepsilon_0}y\right)dy & \text{for } t>\dfrac{x}{c},\\ 0 \text{ for } t<\dfrac{x}{c}.\end{cases}$$

Thus, on the conductor surface

$$Q(0,t)=\mu_0H_0^2\int_0^{\frac{\sigma}{\varepsilon_0}t}I_0^2\left(\frac{\tau}{2}\right)\exp(-\tau)\,d\tau. \tag{26}$$

It follows from Eq. (26) that within the framework of the mathematical model of Eqs. (1), (3), (4), and (24) for any magnetic fields H_0, including superstrong ones $H_0 > \sqrt{\frac{2Q_*}{\mu_0}}$, there exists a finite time interval t_0, defined by the equation

$$\frac{Q_*}{\mu_0H_0^2}=\int_0^{\frac{\sigma}{\varepsilon_0}t_0}I_0^2\left(\frac{\tau}{2}\right)\exp(-\tau)\,d\tau, \tag{27}$$

over the course of which the planar surface of the conductor x = 0 remains conductive (does not undergo a phase transition). For example, for copper at $\sigma = 63\cdot10^6\ (\Omega\cdot\text{m})^{-1}$, $H = (1/3)\cdot H_{min} = 5\cdot10^7$ A/m = 0.62 MOe from Eq. (27) we obtain $\tau_0 \approx 36$ sec.

Figure 1 shows a graph of the function $F_0(z) = \int_0^z I_0^2\left(\frac{\tau}{2}\right)\exp(-\tau)d\tau$. We note that as $z \to +\infty$ $F_0(z) = (1/\pi)\ln z(1 + O(1))$.

Now let $\lambda \neq 0$. Now, not considering the problem of Eqs. (1), (3), (4), and (24) at $\mu_R = \varepsilon_R = 1$ in general form, we find its solution with the assumption that in place of the

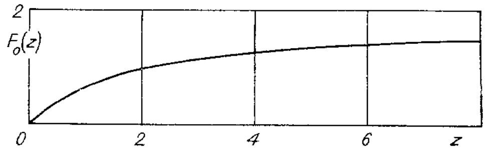

Fig. 1

fifth expression of Eq. 1 we have Eq. (2). It is easily seen that this problem reduces to

$$\frac{\partial^2 q}{\partial t^2} = a^2 \frac{\partial^2 q}{\partial x^2} + f(x, t), \; q(0, t) = 0, \; q(x, 0) = \frac{\partial q}{\partial t}\Big|_{t=0} = 0 \quad (0 < x < \infty)$$

(where $a = \sqrt{\lambda / c_V \tau_0}$ is the heat propagation rate ($a \leq c$), $f(x, t) = -\sigma a^2 \partial E^2(x, t)/\partial x$, $E(x, t)$ are defined by Eq. (25)), the solution of which is well known [5]:

$$q(x, t) = \begin{cases} \frac{1}{2a} \int\limits_0^{t - \frac{x}{c}} \int\limits_{a(t-\tau)-x}^{x+a(t-\tau)} f(z, \tau)\, dz\, d\tau + \frac{1}{2a} \int\limits_{t - \frac{x}{a}}^{t} \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} f(z, \tau)\, dz\, d\tau \text{ for } t > \frac{x}{a}, \\ \frac{1}{2a} \int\limits_0^{t} \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} f(z, \tau)\, dz\, d\tau \text{ for } t < \frac{x}{a}. \end{cases}$$

Using this solution in Eq. (1) and performing certain calculations, we obtain

$$Q(0, t) = \mu_0 H_0^2 \left[\int\limits_{\frac{a}{a+c\varepsilon_0}\frac{\sigma}{} t}^{\frac{\sigma}{\varepsilon_0} t} I_0^2 \left(\frac{1}{2} \sqrt{\tau^2 - \frac{a^2}{c^2} \left(\frac{\sigma}{\varepsilon_0} t - \tau \right)^2} \right) \times \right. \tag{28}$$

$$\left. \times \exp(-\tau)\, d\tau + 1 - \exp\left(-\frac{a}{a+c} \frac{\sigma}{\varepsilon_0} t \right) \right].$$

For solid bodies (metals) $\tau_0 \approx 10^{-11}$ sec [2], therefore, for example, for copper $a \approx 3 \cdot 10^3$ m/sec, $\nu = \frac{a}{c} = \frac{1}{c} \sqrt{\frac{\lambda}{c_V \tau_0}} \approx 10^{-5}$. We note that at $\nu = 0$ from Eq. (28) we have Eq. (26).

Thus we must study the nonnegative function

$$F_\nu(z) = \int\limits_0^z I_0^2 \left(\frac{1}{2} \sqrt{\tau^2 - \nu^2 (z - \tau)^2} \right) \exp(-\tau)\, d\tau \quad (\nu = \text{const})$$

at $z > 0$, where $0 < \nu \leq 1$.

We will present only one result:

$$F_1(z) = 2/\pi z + o(1/z) \text{ for } z \to +\infty. \tag{29}$$

In reality, considering the known asymptote of the Bessel functions [5], we find

$$F_1(z) = \frac{1}{\pi} e^{-\frac{z}{2}} \int\limits_{1/2}^{1} x^{-\frac{1}{2}} e^{\frac{z}{2} x}\, dx + \frac{z}{2} e^{-\frac{z}{2}} \int\limits_0^{\infty} e^{-\frac{z}{2} x} I_0^2 \left(\frac{z}{2} \sqrt{x} \right) dx -$$

$$- \frac{1}{\pi} \int\limits_1^{\infty} e^{-\frac{z}{2} (y-1)^2}\, dy + o\left(\frac{1}{z} \right).$$

Whence, using the known expressions of [3] and integrating the first integral by parts, we have

$$F_1(z) = \frac{2}{\pi z} + \frac{1}{2} I_0 \left(\frac{z}{4} \right) \exp\left(-\frac{z}{4} \right) - \frac{1}{\sqrt{2\pi z}} + o\left(\frac{1}{z} \right)$$

and considering the asymptote $I_0(z/4)$ as $z \to +\infty$, we obtain Eq. (29).

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ANALYSIS OF THE STRESSED STATE OF A SINGLE-TURN BIMETALLIC SOLENOID IN AN INTENSE PULSED MAGNETIC FIELD

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One of the characteristic manifestations of interaction of a pulsed electromagnetic field with conductors is Joulean heating which is nonuniform over the conductor thickness. In the design of electrophysical apparatus using large pulsed currents and magnetic fields one must consider the intense heating of the surface of conductive elements which occurs due to the abrupt surface effect. The high heating temperature is a factor which limits capabilities and uses of equipment since it markedly degrades the strength properties of conductive material, which may lead to large deformation and failure of its conductive elements.

Among the components of high power pulsed electrophysical equipment subjected to the most severe loads are single turn solenoids (Fig. 1) intended for repetitive generation of intense magnetic fields ($B_m \leq 50$ T). In such cases heating of the inner surface reaches hundreds of degrees [1]. The mechanical loads produced by electromagnetic field pondermotor forces can be estimated from the maximum magnetic pressure, equal to the magnetic field energy density in the working volume of the solenoid [2]:

$$P_m = B_m^2/2\mu_0 \tag{1}$$

(where B_m is the induction amplitude, μ_0 is the magnetic constant of a vacuum). Electrodynamic forces are not the only cause of high mechanical stresses in the solenoid. Upon nonuniform heating of the conductor, produced by the abrupt surface effect, thermoelastic stresses develop, which are determined by the gradient of the temperature distribution over thickness. Since the highest temperature is achieved at the end of the field pulse, when the electromagnetic forces are negligibly small, the latter can be neglected in considering the maximum values of the temperature stresses.

For the abrupt surface effect it is simple to obtain an estimate of the thermoelastic stresses by using Lorentz's expression for a long hollow cylinder nonuniformly heated over wall thickness [3]. The azimuthal and axial stresses on the inner cylinder surface can then be written in the form

$$\sigma_z(R_i) = \sigma_\theta(R_i) = \frac{\beta_0 E}{1-\nu}\left[\frac{2}{R_e^2 - R_i^2}\int\limits_{R_i}^{R_e} \Theta r\,dr - \Theta(R_i)\right], \tag{2}$$

where R_i, R_e are the inner and outer radii of the cylinder $\Theta = \Theta(r)$ is the temperature distribution over the cylinder wall thickness, β_0 is the coefficient of linear thermal expan-

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